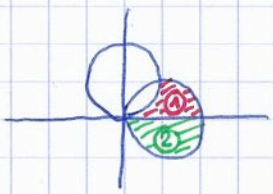
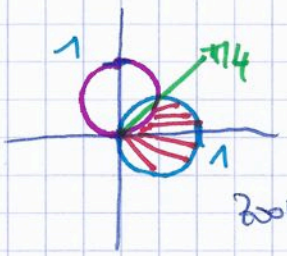
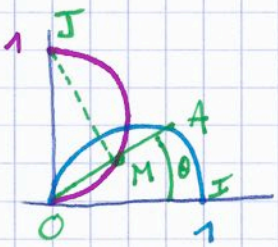
(5) $\iint_D (x^2+y^2)\,dx\,dy$ $\quad D \begin{cases} x^2+y^2-x \le 0 \\ x^2+y^2-y \ge 0 \end{cases} \Leftrightarrow \begin{cases} (x-\frac{1}{2})^2+y^2 \le \frac{1}{4} \\ x^2+(y-\frac{1}{2})^2 \ge \frac{1}{4} \end{cases}$





Zoom



$OM = OJ\cos(\frac{\pi}{2}-\theta)$
$= \sin\theta$
$OA = OI\cos\theta$
$= \cos\theta$

$$I_1 = \int_{\theta=0}^{\pi/4} \int_{r=\sin\theta}^{\cos\theta} r^2 \, r\,dr\,d\theta$$

$$I_1 = \int_{\theta=0}^{\pi/4} \left[\frac{r^4}{4}\right]_{\sin\theta}^{\cos\theta} d\theta$$

astuce $\cos^4-\sin^4 = (\cos^2-\sin^2)\underbrace{(\cos^2+\sin^2)}_{1}$

$$I_1 = \int_{\theta=0}^{\pi/4} \frac{1}{4}(\cos^4\theta-\sin^4\theta)\,d\theta$$

$$I_1 = \int_{\theta=0}^{\pi/4} \frac{1}{4}(\cos^2\theta-\sin^2\theta)\,d\theta$$

$$I_1 = \int_{\theta=0}^{\pi/4} \frac{1}{4}\times\cos(2\theta)\,d\theta$$

$$I_1 = \frac{1}{4}\left[\frac{\sin(2\theta)}{2}\right]_0^{\pi/4} = \frac{1}{8}$$

$$I_2 = \int_{\theta=-\pi/2}^{0} \int_{r=0}^{\cos\theta} r^3\,dr\,d\theta$$

$$= \int_{-\pi/2}^{0} \frac{1}{4}\cos^4\theta\,d\theta$$

$$= \dots = \frac{1}{128}(8+3\pi)$$

1/128 * (9+6π)

~~$I \approx 0{,}261$~~